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IN TYPE II SUPERNOVAE

Hong-Yee Chiu

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NEUTRINO THEORY OF STELLAR COLLAPSE
IN TYPE II SUPERNOVA

Hong-Yee Chiu*

NASA - Goddard Space Flight Center
Institute for Space Studies
New York, N.Y.

and

Sloan Physics Laboratory, Yale University
New Haven, Connecticut

and

Robert W. Fuller

Pupin Physics Laboratory, Columbia University
New York, N.Y.

*Part-time appointment from National Academy of Sciences-National Research Council as Research Associate with the National Aeronautics and Space Administration.

A B S T R A C T

The importance of neutrino processes in the final stage of stellar evolution is further explored. At $T > 3 \times 10^9$ °K the transfer of radiation may be neglected relative to the dissipation of energy due to the pair annihilation process of neutrino production. The equations of stellar interiors are then substantially simplified and physically significant solutions may be obtained analytically by application of the virial theorem. Under conditions prevailing in massive ($M \sim 30\ M_\odot$) population I stars (Type II supernova) at $T \sim 6 \times 10^9$ °K it is found that the rate of stellar contraction due to neutrino emission processes is close to the speed of free fall. This occurs at a temperature below that at which the iron-helium phase change occurs in large scale. Consequently we believe the iron helium phase change never occurs in Type II supernova.

I. Introduction and Summary

It is generally accepted that supernovae represent the final evolutional phase of some stars. To account for the peak luminosity of a supernova ($\sim 10^{10}\ L_{\odot}$)[1] and its total energy output ($\sim 10^{50}$ ergs) it is necessary to consider the energy that gives rise to a supernova explosion to be due to the sudden fusion of a nuclear fuel. This was pointed out qualitatively by the monumental article of Burbidge, Burbidge, Fowler and Hoyle[2] and later discussed in more detail by Hoyle and Fowler[3].

A necessary though not sufficient condition for a nuclear fuel to be explosive is that the fuel must be capable of yielding an adequate energy supply in a time less than the explosive time scale of a star, which is approximately the time for a sound wave to be transmitted across the star. This time scale is around 10-100 sec.

On this premise Hoyle and Fowler excluded many possible nuclear fuels; among these are hydrogen and helium[3]. The time scale for hydrogen burning involves a beta decay time scale of around 300 seconds; this is quite long. The helium reaction (3α reaction) is a three body process and the intermediate product Be^{8} is never present in large quantities. Therefore helium is excluded. As possible candidates for nuclear fuel they include C^{12} and Ne^{20}.

A sufficient condition for potentially explosive nuclear fuel to explode was furnished by Hoyle in 1946[4]. His mechanism is based on the properties of elements in statistical equilibrium at high temperatures. His idea is as follows: when the temperature T is below a certain critical temperature T_{cr} (which is a function of density, but in general is of the order of 7 BK (billions of degrees K) the equilibrium configuration is such that over 90% of the constituents is Fe^{56} which has the highest binding energy (around 8.4 Mev per nucleon)). This equilibrium configuration (Fe^{56}) is reached at a temperature of around 4.5 BK according to a recent estimate by A. G. W. Cameron[5]. When the temperature T exceeds T_{cr}, the equilibrium configuration for the most abundant element suddenly changes from Fe^{56} to He^4 (~ 90% at a temperature somewhat greater than T_{cr}) which has a binding energy of around 6.8 Mev per nucleon. Inside a star the center is hottest and it is therefore safe to assume the transition takes place somewhere inside the central core of a star. Since the core is already in statistical equilibrium, there will not be nuclear reactions to provide the energy for iron-helium phase change, and gravitational contraction is the only energy source left. At this temperature the thermal energy is around 3×10^{17} ergs/g whereas the $Fe^{56} \rightarrow He^4$

transition will require 1.5×10^{18} ergs/g. Therefore the transition will take place only at the expense of a gravitational contraction -- and such contraction will not raise the temperature of the medium until the transition is completed. This will cause great mechanical instability and the central core essentially collapses. This means the time scale for evolution of the central core is comparable to the time scale for free fall toward the center, so that hydrostatic equilibrium loses its meaning in such a rapid evolutional phase.

In the outer region where nuclear fuel is still available, a collapse in the center necessarily raises the temperature rapidly -- and because of the steep dependence of nuclear reaction rates on temperature (the temperature dependence is roughly $\exp\left(\frac{85}{T_9^{1/3}}\right)$ (3) a potentially explosive nuclear fuel will quickly release all its energy, thus producing an explosion.

Hoyle and Fowler divide pre-supernova stars into two categories corresponding to the two types of supernovae. The pre-supernova star that gives rise to Type II supernova (mostly found in the arms of galaxies) is very massive ($M \sim 30\ M_\odot$) and the center of such stars is never degenerate. This paradoxical fact was explained in their original paper.(3)

In such cases the implosion takes place and causes explosion. Type I supernovae are less massive and the center is quite degenerate. The implosion never really takes place and the nuclear fuel undergoes sudden fusion by a process described in Ref. (3).

In this paper we shall pursue the problem of the structure of pre-supernova stars of Population I, Type II supernovae in some detail, including the effect of the recently discussed annihilation process of neutrino production which has a higher neutrino emission rate than any other known processes of neutrino production in stars[10]. Using the model of Hoyle and Fowler we get different results from theirs as to the cause of instability of a Population I, Type II supernova star. However, their main conclusions concerning nucleosynthesis remain unchanged. We find that the temperature at which iron-helium conversion takes place is not reached before the explosion takes place in the nuclear fuel rich region.

II. Neutrino Process

A number of neutrino processes have been investigated.[6-11] The relative importance of these processes has been summarized in Ref.(11). Here we shall concern ourselves with the annihilation process of neutrino production:[10]

$$e^- + e^+ \to \nu + \bar{\nu} \tag{2.1}$$

which have been shown to be most important in the temperature regime 10^9 °K - 10^{10} °K. A machine calculation of the rate of energy loss due to process (2.1) has been performed[12].

In (2.1) the electron-positron pair is created in equilibrium with thermal radiation. Fig. 1 and Fig. 2 reproduce the result of Ref.(12). For a non-degenerate medium the rate of energy loss $\frac{dU_\nu}{dt}$ (in ergs/cm^3- sec) is approximately given as[11]

$$-\frac{dU_\nu}{dt} = 4.8 \times 10^{18}\ T_9^{\,3} \exp\left(-\frac{11.9}{T_9}\right),\ T_9 \ll 6 \tag{2.2}$$

$$-\frac{dU_\nu}{dt} = 4.3 \times 10^{15}\ T_9^{\,9}\ ,\quad T_9 \gg 6 \tag{2.3}$$

where $T = \frac{T}{10^9\ {}^\circ K}$ and U is the energy density. The subscript ν is used to indicate the fact that this energy is lost to neutrinos. The effect of degeneracy on (2.2) and (2.3) is complicated (Fig. 2). However, when $T \sim 5 \times 10^9$ °K, $\rho \sim 10^7$ g/cm^3 degeneracy is not important[12].

At T = 5 BK an empirical formula (extracted from Fig. 1) which describes the neutrino process (2.1) to a satisfactory degree of accuracy is:

$$-\frac{dE_\nu}{dt} = \frac{7.4 \times 10^{21}}{\rho}\left(\frac{T}{5 \times 10^9\ {}^\circ K}\right)^{9.2} \text{ergs/g-sec} \tag{2.4}$$

where E is the energy per gram of matter, and the subscript ν has the same meaning as above.

III. Basic Equations

The basic equations of stellar structure are:

$$\frac{dP}{dr} = -\rho \frac{GM}{r^2} \tag{3.1}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \tag{3.2}$$

$$\frac{dL}{dr} = 4\pi r^2 \rho \,\mathcal{E} \tag{3.3}$$

$$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\kappa\rho}{T^3} \frac{L}{4\pi r^2} \qquad \text{(radiative)} \tag{3.4}$$

where P is the pressure, ρ the density, M the mass that is contained inside the sphere of symmetry of radius r, L is the total flux of energy that passes through the spherical shell of radius r, $\mathcal{E}$ is the net energy produced per unit mass at r, κ is the opacity per unit mass. The rest of the symbols have their usual meaning (all symbols are defined in Appendix I.) Typical solutions of Eqs. (3.1)-(3.4) may be found in standard text books[13].

Eq.(3.1) describes the structure of a star in hydrostatic equilibrium and Eq.(3.2) essentially defines M. Eqs.(3.3) and (3.4) need special discussion.

Eq.(3.3) has been extensively discussed[14]. It has been argued that the energy that is lost to neutrinos is several orders of magnitude greater than the observed luminosity of stars. Therefore, we are justified in neglecting the radiation energy produced per cm^3, $\frac{dL}{dM} = \frac{dL}{dr}\frac{1}{4\pi r^2 \rho}$; the value of $\mathcal{E}$ is then zero. $\mathcal{E}$ assumes the following form:

$$\mathcal{E} = -\frac{Pdv}{dt} - \frac{dE}{dt} + \frac{dE_\nu}{dt} = 0 \qquad (3.5)$$

where v is the volume occupied by a unit mass, E is thermal energy per unit mass; the term $-\frac{Pdv}{dt} - \frac{dE}{dt}$ is the rate at which energy is provided by gravitational contraction; $-\frac{dE_\nu}{dt}$ is the rate at which energy is lost to neutrinos. For further details see Ref.(14).

Eq.(3.4) describes the transfer of radiative energy inside stars. The importance of radiative energy transfer may be estimated as follows: if, during the whole life of a star the amount of radiative energy that flows into a given volume is small compared to its total thermal energy content, then the role played by radiative energy transfer is unimportant. Usually the total amount of radiative energy that flows into a given volume in the total life time of a star is large compared to its thermal energy content. But in the case where the pair annihilation neutrino process plays a principal role, this is not true as we shall show below.

At $T = 5$ BK, $\rho = 10^7 g/cm^3$ (these values are consistent with the estimate of Hoyle and Fowler for Population I pre-supernova star)[3], the radiative energy f that flows across a unit area in time θ is given by

$$f = \int_0^\theta \frac{L}{4\pi r^2} dt \approx \frac{L}{4\pi r^2} \theta \tag{3.6}$$

From Eqs.(3.4) and (3.6) we have

$$f \simeq \frac{L\theta}{4\pi r^2} = -\frac{4ac}{3} \frac{T^3}{\kappa\rho} \frac{dT}{dr} \theta \tag{3.7}$$

Assuming the source of opacity is due to electron Thomson scattering, κ assumes the value 0.19[15]. Other sources of opacity are relatively unimportant[16]. $\frac{dT}{dr}$ may be estimated as follows: the region where the neutrino process is dominant may contain masses up to $20M_\odot$; the density may be taken to be 10^7 g/cm^3; the radius is around 10^9 cm ; the temperature at the center is taken to be 5 BK. Therefore, the average temperature gradient is around

$$-\left(\frac{dT}{dr}\right)_{AV} = \frac{5\ \text{BK}}{10^9\ \text{cm}} = 5\ {}^\circ\text{K/cm} \tag{3.8}$$

Putting the numerical values of the various quantities into Eq.(3.7) we obtain

$$f = \frac{L}{4\pi r^2}\,\theta = 1.59 \times 10^{19}\, \frac{T_9{}^3}{\rho_8} \left|\frac{dT_9}{dr_8}\right| \theta \text{ (ergs/cm}^2\text{)} \tag{3.9}$$

where

$T_9 = \frac{T}{10^9\ {}^{o}K}$, $\rho_8 = \frac{\rho}{10^8}$ g/cm^3, r_8 = r/10^8 cm. We may remark that in general $\rho \propto T^3$ and f/θ will not be changed very much in the course of stellar evolution.

The energy that is transmitted (E_{TRANS}) into a unit volume is, for $\rho_8 = 10$, $T_9 = 5$, $r_8 = 10$,

$$E_{TRANS} \cong 10^{20}\ \theta \text{ ergs/cm}^3 \tag{3.10}$$

The total energy content is

$$\begin{aligned} E_{INT} &\approx E_{GAS} + E_{RAD} \\ E_{GAS} &\cong 3NkT/\text{cm}^3 = 6.23 \times 10^{24} \text{ ergs/cm}^3 \qquad (3.11) \\ E_{RAD} &= aT^4 = 4.72 \times 10^{24} \text{ ergs/cm}^3 \\ T &= 5 \times 10^9\ {}^{o}K \\ E_{INT} &\approx 1.1 \times 10^{25} \text{ ergs/cm} \\ \theta &\cong 10^5 \text{ sec} \end{aligned}$$

where N is the electron number density

$$N = \frac{\rho}{\mu_e m_p}\ ; \qquad \mu_e = \frac{\langle A \rangle}{\langle Z \rangle} \approx 2$$

Later we shall demonstrate, assuming that radiative transfer may be neglected, that catastrophic explosion will occur in a time close to 100 seconds at T = 5 BK. Therefore neglecting the radiative energy transfer is self-consistent.

We now state the vivial theorem which follows from Eqs. (3.1) and (3.2)[17]. It will be used in deriving the total energy of the star (Eq.(3.14)).

$$\int 3PdV = \int \frac{GMdM}{r} = -E_{gr} \tag{3.13}$$

The total energy of the star (E_{tot}) is

$$E_{tot} \equiv E_{gr} + E_{int} = -\int 3PdV + \int E\rho dV \tag{3.14}$$

$$= -\int \left(\frac{3P}{E\rho} - 1\right) E\rho dV$$

where E is the internal energy per unit mass (in our case E is nearly all thermal energy), and $E\rho = U$, the thermal energy per unit volume $\left(\frac{3P}{E\rho} - 1\right)$ is usually between 0 and 1. The numerical value of $\left(\frac{3P}{E\rho} - 1\right)$ at $T = 5 \times 10^{9}$ °K, $\rho = 10^{7}$ g/cm^{3} is less than 0.1 (See Appendix II).

Now the star may be divided roughly, but unambiguously, into two parts: the core and the envelope. Inside the core the neutrino process is dominant, whereas in the envelope the neutrino process is relatively unimportant.

The total energy of a star decreases when the star radiates energy. In the outer envelope where nuclear energy is still available, the radiated energy comes from the burning of nuclear fuel and gravitational energy is not released to maintain equilibrium. In the core this is not so. The only energy source is gravitational contraction, since in the core all the nuclear fuel has already burned. The rapid dissipation of energy through the neutrino process (2.1) causes the core to contract rapidly, thus decreasing E_{TOT} . Gravitational energy is released through the decrease of E_{TOT}, and is radiated away as neutrinos.

Therefore, it is logical to write:

$$-\frac{d}{dt} E_{TOT} = -\int \frac{dU_\nu}{dt} dV \tag{3.15}$$

where $-\frac{dU_\nu}{dt}$ is the rate of energy loss due to neutrinos per unit volume. From Eq.(3.14) and (3.15) we have

$$\frac{d}{dt} \int \left(\frac{3P}{E\rho} - 1\right) E dM = - \int \frac{dU_\nu}{dt} dV \tag{3.16}$$

As we shall see in Appendix II, $\frac{3P}{E\rho}$ is not sensitive to T and ρ (and consequently t, the time) in the temperature and density regime we consider. We may therefore write

$$\int \left(\frac{3P}{E\rho} - 1\right) \frac{dE}{dt} dM = -\int \frac{dE_\nu}{dt} dM \tag{3.17}$$

(Note: $-\frac{dU_\nu}{dt} = -\rho \frac{dE_\nu}{dt}$ and $\rho dV = dM$ by definition)

Since all radiative transfer processes are negligible as compared to the neutrino process, we equate the integrals for the same spherical shell of mass. Therefore we have the following equation:

$$\left(\frac{3P}{E\rho} - 1\right) \frac{dE}{dT} \left(\frac{dT}{dt}\right)_M = - \left(\frac{dE_\nu}{dt}\right)_M \tag{3.18}$$

Eq.(3.18) replaces Eq.(3.4). The subscript M emphasizes the fact that the left hand and the right hand side refer to the same shell mass element.

In deriving Eq.(3.18) we have neglected the role played by the envelope and also the kinetic energy imparted to the contracting material of the star. We shall review these points later.

IV. General Solution of the Basic Equations

The equation of radiative transfer (3.4) has been replaced by the following equation (3.18).

$$\left(\frac{3P}{E\rho}-1\right)\left(\frac{dE}{dT}\right)\left(\frac{dT}{dt}\right)_M = - \left(\frac{dE_\nu}{dt}\right)_M \tag{4.1}$$

In Sect. II we obtain the form of $-\frac{dE_\nu}{dt}$ as

$$-\frac{dE_\nu}{dt} = \frac{A}{\rho} \left(\frac{T}{T_\nu}\right)^n \text{ ergs/gm-sec} \tag{4.2}$$

where A and n are constants:

$$A = 7.4 \times 10^{21}$$

$$N = 9.2 \tag{4.3}$$

$$T_\nu = 5 \text{ BK}$$

This equation may be viewed as a generalized Stefan-Boltzmann equation for neutrino emission. The value of N approaches 9 in the relativistic limit ($KT > mc^2$).

Write $\frac{dE}{dT} \approx C_v$, $\left(\frac{3P}{E\rho} - 1 \right) \equiv \eta$ (4.4)

Then Eq.(4.1) becomes

$$- \frac{dE_\nu}{dt} = \eta C_v \frac{dT}{dt} = \frac{A}{\rho} \left(\frac{T}{T_\nu}\right)^n \tag{4.5}$$

The subscripts M have been dropped—they are to be understood in the sequel.

Eq.(4.5) may be coupled with Eq.(3.5) to solve for T and ρ as functions of t for constant M. Eq.(3.5) is

$$- \left(P\frac{dv}{dt} + \frac{dE}{dt} \right) + \frac{dE_\nu}{dt} = 0 \tag{3.5}$$

Since v is the volume occupied by a unit mass, $v = \frac{1}{\rho}$

Thus, Eq.(3.5) becomes

$$\frac{P}{\rho^2} \frac{d\rho}{dt} - \frac{dE}{dT}\frac{dT}{dt} - \frac{A}{\rho} \left(\frac{T}{T_\nu} \right)^n = 0 \tag{4.6}$$

Substituting Eq.(4.5) into Eq.(4.6) we have

$$\frac{P}{\rho^2} \frac{d\rho}{dt} - (1 + \eta) C_v \frac{dT}{dt} = 0 \tag{4.7}$$

We write $P = \rho RT$. Then Eq.(4.7) becomes:

$$\frac{RT}{\rho} \frac{d\rho}{dt} - (1 + \eta) C_v \frac{dT}{dt} = 0 \tag{4.8}$$

The solution of Eq.(4.8) is:

$$\frac{\rho(M,t)}{\rho(M,0)} = \left(\frac{T(M,t)}{T(M,0)} \right)^{(1+\eta)C_v/R} \tag{4.9}$$

where $\rho(M,0)$ and $T(M,0)$ are the initial distributions of ρ and T as functions of M at $t = 0$. The exponent $\frac{(1+\eta)c_\nu}{R} \equiv \alpha$ is close to 3, since

$$c_\nu = 2.7R \qquad \eta \cong 0.1 \tag{4.10}$$

The value of c_ν is from a table prepared by Chandrasekhar[18].

Substituting Eq.(4.9) into Eq.(4.5) we obtain:

$$\eta C_v \frac{dT}{dt} = \frac{A(\frac{T}{T_\nu})^n}{\rho_o(\frac{T}{T_O})^\alpha}$$

$$\frac{dT}{dt} = \frac{A}{T_\nu n} \frac{(T\ (0))^\alpha}{\rho(0)} \frac{1}{\eta C_v} T^{n-\alpha} \tag{4.11}$$

Let
$$g \equiv \frac{A}{T_\nu^n} \frac{(T(0))^\alpha}{\rho(0)} \frac{1}{\eta C_v} \tag{4.12}$$

then
$$\frac{dT}{dt} = gt^{n-\alpha} \tag{4.13}$$

The solution of Eq.(4.13) is

$$\frac{T^{-(n-\alpha-1)}}{-(n-\alpha-1)} = gt + \text{const.} \tag{4.14}$$

At $t = 0$, $T = T\ (0)$; so Eq.(4.14) becomes

$$T = \frac{1}{\left\{\left[\frac{1}{T(0)}\right]^{n-\alpha-1} - (n-\alpha-1)gt\right\}^{1/(n-\alpha-1)}} \tag{4.15}$$

$$= \frac{T(0)}{\left\{1-(n-\alpha-1)[T(0)]^{n-\alpha-1}gt\right\}^{1/(n-\alpha-1)}}$$

Substituting g into the coefficient of t in Eq.(4.15) we have

$$B \overset{=}{=} (n-\alpha-1)\ [T(0)]^{(n-\alpha-1)} g$$
$$= (n-\alpha-1)(T(0))^{n-\alpha-1} \frac{A}{T_\nu^{\ n}} \frac{(T(0))^{\alpha}}{\rho(0)} \frac{1}{\eta C_v}$$
$$= (n-\alpha-1) \frac{A}{\rho(0)} \left(\frac{T(0)}{T_\nu}\right)^n \frac{1}{\eta C_v T(0)} \tag{4.16}$$

Now $\quad C_v T(0) \approx E\ (T(0))$

and $$-\left(\frac{dE_\nu}{dt}\right)_0 = \frac{A}{\rho(0)} \left(\frac{T(0)}{T_\nu}\right)^n \tag{4.17}$$

Define $$\tau \equiv E/\left|\frac{dE_\nu}{dt}\right| \tag{4.18}$$

τ is the relaxation time for cooling by neutrinos. Then Eq.(4.16) becomes:

$$B = \frac{(n-\alpha-1)}{\eta} \frac{1}{\tau} \tag{4.19}$$

Define $\quad t_\infty = \frac{1}{B}$ so that

$$T = \frac{T(0)}{\left(1 - \frac{t}{t_\infty}\right)^{1/(n-\alpha-1)}} \tag{4.20}$$

$$t_\infty = \frac{\eta}{(n-\alpha-1)} \tau$$

From Ref.(12), (also see Sect. II) $\rho = 6 \times 10^{6}$ g/cm^3, at $T_9 = 5$, $\tau \cong 10^3$ sec. $\eta \approx 0.05$ (See Appendix II) $(n - \alpha - 1) \cong 5$, thus $t_\infty = 0.01\,\tau \cong 10$ sec and at $T_9 = 6.9$, $\rho = 10^7$ g/cm^3, $\tau \cong 100$ sec, $t_\infty \cong 1$ sec.

Mathematically $T = \infty$ at $t = t_\infty$. It is very tempting to assign t_∞ as the time scale for stellar collapse. But we may not do this because Eq.(4.20) has been obtained with the assumption that the hydrostatic equilibrium condition (Eqs.(3.1) and (3.2)) are strictly valid. We have neglected the radiation pressure and used a number of other approximations.

Physically t_∞ may be assigned as the characteristic time for the evolution of the core. When t approaches t_∞ the kinetic energy of the contracting mass of the core is already so great that the condition of hydrostatic equilibrium (Eqs.(3.1) and (3.2)) do not provide an adequate description. The kinetic energy comes from gravitational contraction so that we must add another term on the right hand side of Eq.(3.18) and Eq.(3.1) to take care of it. In the case of Eq.(3.18) this term effectively increases the rate of contraction and the effect is to make t_∞ even smaller--which means the contraction process proceeds even faster.

What is the rate of contraction of the core? To estimate it we need to work out both the spatial and the time dependence of ρ and T, from which we may obtain the rate of shrinkage of the core. We shall solve a particular model in the next section.

V. The Contraction Rate of a Specific Model

We shall use the model for Type II supernova studied by Hoyle and Fowler[3]. Their work is based on the following assumptions:

1) The mass of the star is divided between the core and the envelope in the ratio 2:1.

2) The core, with nearly constant μ_E, possesses a structure similar to that of main sequence stars of uniform composition, known to be similar to that of the standard model[19].

Then
$$\rho = \frac{a\mu_E \beta}{3R(1-\beta)} T^3 \tag{5.1}$$

$$1 - \beta = 0.0030 \left(\frac{2M}{3M_\odot}\right)^2 \mu_E^4 \beta^4 \tag{5.2}$$

β is the ratio of gas pressure to total pressure. In the work of Hoyle and Fowler[3] $\mu_E = 2.1$. Eqs.(5.1) and (5.2) are taken from Ref.(3).

From Eq.(5.2) with $M = 30\ M_\odot$ they obtained $\beta \cong 0.4$. Eq.(5.1) becomes

$$\rho = 4.3 \times 10^4\ T_9^{\ 3}\ \mathrm{g/cm^3} \tag{5.3}$$

We now compute the rate of contraction of the core with the parameters given by their model. Eq.(5.3), together with Eqs.(3.1) and (3.2), allows us to solve for ρ and T as functions of r. This involves the use of the solution of the standard Lane-Emden equation of index n = 3[20].

The solutions are:

$$P = \left[\left(\frac{k}{\mu_E m_p} \right)^4 \frac{3}{a} \frac{1-\beta}{\beta^4} \right]^{1/3} \rho^{4/3} \equiv K\rho^{4/3} \tag{5.4}$$

$$\rho(\xi,t) - \rho_c(t)\theta_3^{\,3}(\xi) \tag{5.5}$$

$$T(\xi,t) = T_c(0)\theta_3(\xi) \tag{5.6}$$

$$r(t) = \alpha(t)\xi \tag{5.7}$$

$$\alpha(t) = \left[\frac{(n+1)K}{4\pi G} \right]^{\frac{1}{2}} [\rho_c(t)]^{\frac{1-n}{2n}} \tag{5.8}$$

n = polytropic index, (= 3 in our case)

where $\theta_3(\xi)$ is the Lane-Emden function of index 3 normalized so that $\theta_3(0) = 1$. $\theta_3(\xi)$ has a zero at $\xi = \xi_1 \approx 6.9$ which is taken to be the natural boundary of the core. From a table prepared by Chandrasekhar[21], for $\beta = 0.4$, $\left(\frac{M}{M_\odot}\right) \mu_E^2 = 87.04$ at $\xi = \xi'$. For $\mu_E = 2.1$ the mass contained in the core is around 20 $M_\odot$ so that the radius of the core is $R_c = \alpha\xi' \cong 6.9\alpha$. The mass of the core determined in this way equals $2/3(30M_\odot)$ which is consistent with assumption (1).

From Eq.(4.9):

$$\frac{\rho(M,t)}{\rho(M,0)} = \left[\frac{T(M,t)}{T(M,0)} \right]^{\frac{(1+\eta)C_v'}{R}} \tag{4.9}$$

and from the numerical values of η and C_v' and Eq.(4.10) we find the exponent $(1+\eta)C_v'/R$ is very close to 3. Eq.(4.9) with this exponent is the same equation used by Hoyle and Fowler. This model star therefore contracts homologously. In the log ρ-log T plane the initial structure is a straight line of slope 3. The maximum abscissa and ordinate of the line are determined by the values of ρ and T at the center of the star. Later as time elapses ρ_c and T_c will increase according to Eq.(4.9) and Eq.(4.20).

Eq.(4.9) and (5.3) are of the same form. Therefore in Eqs.(5.5) and (5.6) the time dependence appears in ρ_c and T_c only. The radius of the core $R_c \cong 6.9\alpha$. The time rate of change of R_c is then

$$\frac{dR_c}{dt} = 6.9 \frac{d\alpha}{dt}$$

Assuming β, μ_E are not rapidly varying functions of t, then

$$\frac{d\alpha}{dt} = -\frac{1}{3}\left\{\frac{K}{\pi G}\right\}^{\frac{1}{2}} \rho_c^{-4/3} \frac{d\rho_c}{dt}$$

$$= -\frac{1}{3}\left\{\frac{K}{\pi G}\right\}^{\frac{1}{2}} \rho_c^{-1/3} \frac{d\ln\rho_c}{dt}$$

Numerically

$$K = [(\frac{k}{\mu_E M_P})^4 \frac{3}{a} \frac{1-\beta}{\beta^4}]^{1/3} = 2.8 \times 10^{15} \tag{5.11}$$

$$G = 6.67 \times 10^{-8}$$

$$\frac{d\alpha}{dt} = -0.387 \times 10^{11} \rho_c^{-1/3} \frac{d\ln\rho_c}{dt} \tag{5.12}$$

$$\frac{dR_c}{dt} = -6.9 \frac{d\alpha}{dt} = -2.7 \times 10^{11} \rho_c^{-1/3} \frac{d\ln\rho_c}{dt} \tag{5.13}$$

For

$$\rho_c = 10^7 \ g/cm^3$$

$$\frac{dR_c}{dt} = -1.25 \times 10^9 \frac{d\ln\rho_c}{dt} \tag{5.13a}$$

From Eqs.(4.20) and (4.9) we have

$$\frac{d\ln\rho_c}{dt} = \frac{\frac{3}{n-\alpha-1}}{t_\infty} \frac{1}{(1-\frac{t}{t_\infty})} \tag{5.14}$$

At $T = 5 \times 10^9$ °K, $t_\infty = 10$ sec and the contraction rate of the core is close to 7.5×10^7 cm/sec at $t = 0$ (as has been remarked in Sect. IV, this method calculation will give the most reliable results near $t = 0$). At $T = 6.9 \times 10^9$ °K, the critical temperature for iron-helium conversion, $t_\infty = 1$ sec, the contraction rate of the core at $t = 0$ is 7.5×10^8 cm/sec.

The radius of the core R_c is $\alpha\xi_1 = 6.9\alpha$. Numerically at $\rho_c = 10^7$ g/cm³

$$R_c = 6.9 \times \left(\frac{K}{\pi G}\right)^{1/2} \left[\rho_c(t)\right]^{-1/3} = 3.7 \times 10^9 \text{ cm} \tag{5.15}$$

The contracting time scale, τ_{CONT}, defined as

$$\tau_{CONT} = \frac{R_c}{\left|\frac{dR_c}{dt}\right|} \tag{5.16}$$

is around 50 seconds at $T = 5 \times 10^9$ $^{\circ}$K and only 5 seconds at $T = 6.9 \times 10^9$ $^{\circ}$K. This is comparable to the free fall time t_{FF} which is given by

$$t_{FF} = \frac{\pi}{2}\left[\frac{R_c^3}{2GM_c}\right]^{1/2} \tag{5.17}$$

For our case, $R_c = 3.75 \times 10^9$ cm, $M_c = 20\ M_{\odot}$, $t_{FF} \cong 5$ sec. This calculation demonstrates that at $T = 6.9 \times 10^9$ $^{\circ}$K a condition very similar to free fall conditions obtains. This free fall condition is initiated by the rapid energy loss due to neutrino emission. Obviously a better computation may be made starting at lower temperatures (say, 2 BK) taking into account the kinetic energy of the contracting mass, and the possible transfer of radiation energy, and following the evolution of the core to the "implosion" point. Whatever these refinements may add, we believe nuclear fuel capable of undergoing fusion in a short time scale and will start to explode in the envelope somewhat before the temperature is reached in the core at which the iron-helium conversion occurs in large scale. The resulting explosion is believed to be a supernova.

VI. Discussions

In their paper[3] Hoyle and Fowler stated that the collapse of the core is due to the iron-helium phase change. We have found, at the temperature at which the phase changes, the core is already collapsing because of energy dissipation by neutrino emission. In our calculation we have neglected the part of the gravitational energy that becomes kinetic energy of the contracting mass. If this kinetic energy is taken into account the free fall condition will be achieved at a temperature even lower than 6.9×10^{9} °K. After the supernova is formed, we are not certain to what degree the iron-helium conversion may have proceeded, since it is not a necessary condition for collapse*.

This calculation indicates that a proper theory for presupernova stars should include the dynamics of a contracting core due to the neutrino process.

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* A. G. W. Cameron (private communication) suggested a model for population I presupernova stars in which the central density at $T = 6.9 \times 10^{9}$ °K is around 10^{9} g/cm^{3}. In such a model the cause of stellar collapse would be entirely due to iron-helium conversion.

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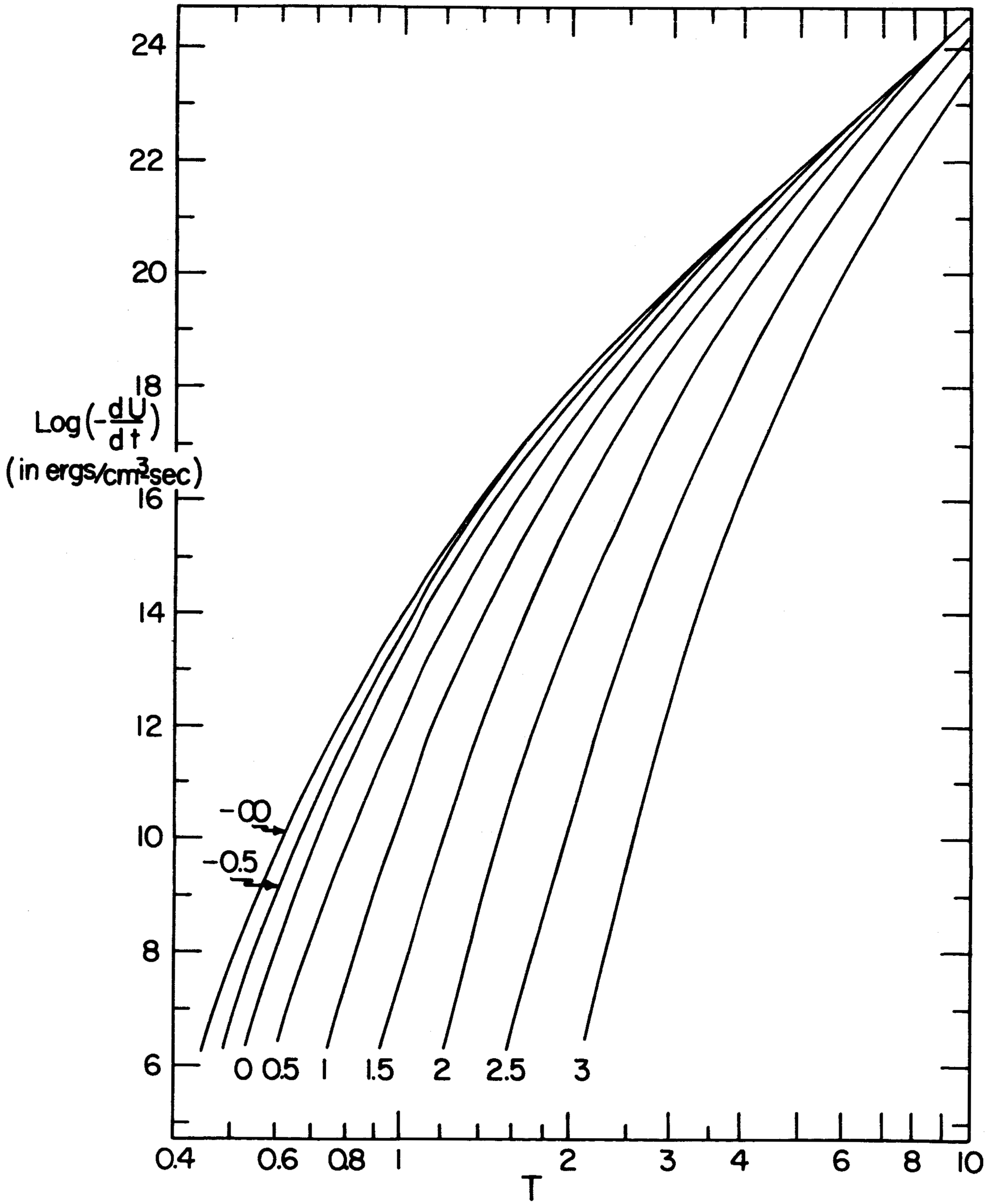
Figure Captions

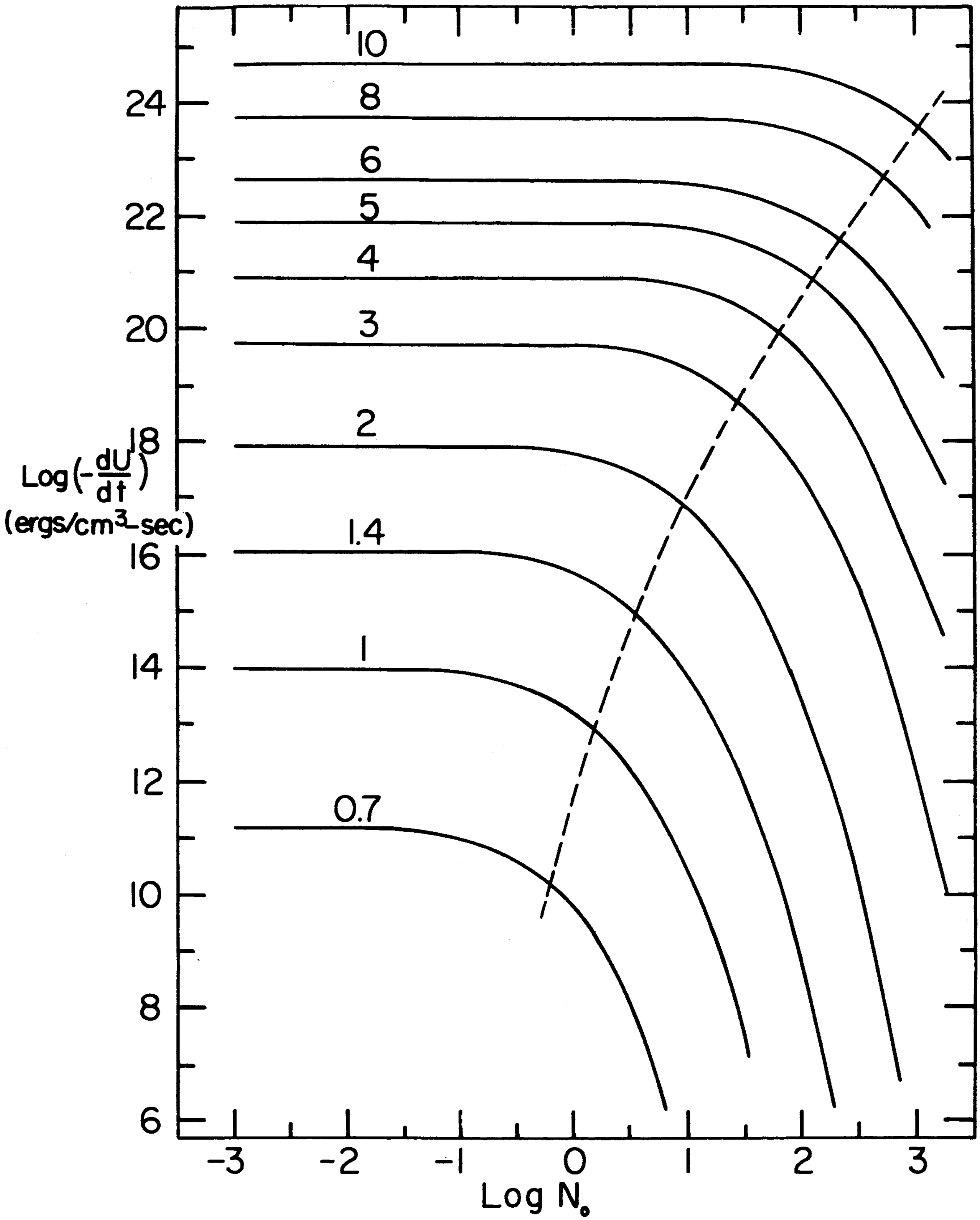
Fig. 1. The rate of energy loss due to the annihilation process (2.1) $\log_{10}\left(-\frac{dU_\nu}{dt}\right)$ is plotted against T for different values of $\log_{10}N_o$, where N_o is the number density for electrons excluding pairs (Ref.(12)). $\log_{10}N_o = 0$ corresponds to a matter density of $3 \times 10^8\ \mu_e\ g/cm^3$ where μ_e is the molecular weight for electrons. $\frac{dU_\nu}{dt}$ is measured in units of $ergs/cm^3$-sec and T in units of BK. Numbers attached to curves are values of $\log_{10}N_o$.

Fig. 2. The rate of energy loss of (2.1) as a function of density. $\log_{10}\left(-\frac{dU_\nu}{dt}\right)$ is plotted against $\log_{10}N_o$ for different values of T_9. T_9 is T measured in BK.



24
22
20
18
16
14
12
10
8
6
Log(-dU/dt)
(in ergs/cm³sec)
-∞
-0.5
0
0.5
1
1.5
2
2.5
3
0.4
0.6
0.8
1
2
3
4
6
8
10
T

Fig. 1



10
8
6
5
4
3
2
1.4
1
0.7
24
22
20
18
16
14
12
10
8
6
Log(-dU/dt)
(ergs/cm3-sec)
-3
-2
-1
0
1
2
3
Log N₀

Fig. 2

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APPENDIX I -- TABLE OF NOTATIONS

a	Stefan-Boltzmann Constant (= 7.55×10^{-15} ergs/$(^{\circ}K)^4$-sec)
A	mass number of a nucleus
α	$\dfrac{(1+\eta)C_V}{R}$
β	ratio of gas pressure to total pressure.
B	See Eq.(4.16)
BK	= 10^9 $^{\circ}$K (one <u>B</u>illion degrees <u>K</u>elvin)
c	light velocity (= 3×10^{10} cm/sec)
C_V	specific heat at constant volume
$\dfrac{dE}{dt}$	the rate of neutrino energy loss in ergs per gram per sec
E_{tot}	the total energy of a star (in ergs)
E_{gr}	the total gravitational energy of a star
E	the internal energy (including thermal energy and Fermi energy) in ergs per gram
E_{int}	the total internal energy of the star
$\mathcal{E}$	rate of net energy release per gram of stellar matter
f	integrated radiative energy flux from 5 = 0 to t = θ
g	See Eq.(4.13)
G	gravitational constant (= 6.67×10^{-8} d cm^2 g^{-2})
k	Boltzmann constant
K	See Eq.(5.4)
κ	opacity coefficient

L	electromagnetic luminosity (total electromagnetic energy flux of a star) (in ergs/sec)
$L_\odot$	solar luminosity (= 3.78×10^{33} ergs/sec)
M	mass inside a shell of radius r
M_p	mass of protons
$M_\odot$	solar mass (= 1.985×10^{33} g)
μ_e	average number of nucleons per electron
n	polytropic index
η	$\frac{3P}{E\rho} - 1$
P	pressure
ρ	density (in g-cm^{-3})
$\rho(0)$	initial value of density
R	gas constant (= 8.32×10^{-7} ergs mole^{-1} deg^{-1})
t	time
T	temperature (in $^\circ$K)
T_{cr}	critical temperature for iron-helium phase change
T_9	T in units of BK
$T(0)$	initial values of T
τ	relaxation time for cooling by neutrinos. See Eq. (4.18)
τ_{cont}	contraction time scale. See Eq. (5.16)
U	energy density (in ergs cm^{-3})
$\frac{dU_\nu}{dt}$	rate of neutrino energy loss per cm^3 per sec
V	specific volume (volume occupied by a unit of mass = $1/\rho$) (in cm^3 g^{-1})
$\mathcal{V}$	volume (in units of cm^3)
Z	atomic number

APPENDIX II

Here we tabulate a few numbers for $\left(\frac{3P}{E\rho} - 1\right) \equiv \eta$ as a function of T. The density is taken from the equation

$$\rho = 4.3 \times 10^4 \, T_9^3 \ \text{g/cm}^3 .$$

T_9	$\log_{10}\rho$	$\eta = \left(\frac{3P}{E\rho} - 1\right)$
1	4.6332	0.675
2	5.5362	0.180
3	6.0654	0.092
4	6.4392	0.024
5	6.7323	0.027
6	6.9684	0.034
7	7.1682	0.035

$\frac{3P}{E\rho}$ is obtained as follows: P includes the radiation pressure and electron pressure (including pairs) and $E\rho = U$ is the energy density of radiation, electron pair energy (including their rest energy) and the electronic energy (less the rest energy). The reason we include the rest energy of pairs to the total energy density is the energy of creating pairs is supplied by the gravitational contraction. The values of P and $E\rho$ are taken from a machine calculation of energy and pressure integrals of an electron gas and radiation in equilibrium. This calculation (performed by S. Tsuruta) is to be published.

In the above table the contribution of heavy nuclei is not included since they do not alter the value of $\frac{3P}{E\rho}$ by more than 2%, and that of η by more than 0.02.